Differentiation of Log Functions

The logarithmic function is the inverse of the exponential function. The natural logarithm $\log_e x$ can be written as $\ln x$.

If $y = \ln x$, then $y' = \frac{1}{x}$

If $y = \ln[f(x)]$, then $y' = f'(x) \cdot \frac{1}{f(x)} = \frac{f'(x)}{f(x)}$

Log rules:
- $\log a^b = b \log a$
- $\log \frac{a}{b} = \log a - \log b$
- $\log ab = \log a + \log b$

Example: Differentiate these log functions.

1) $y = \ln(6x)$ $\quad y' = \frac{6}{6x} = \frac{1}{x}$

2) $y = \ln(3x + 2)$ $\quad y' = \frac{3}{3x+2}$

3) $y = \ln(x^3)$ $\quad y' = \frac{3x^2}{x^3} = \frac{3}{x}$

4) $y = \ln(\sqrt{3x + 2})$ $\quad y = \ln(3x + 2)^{1/2} = \frac{1}{2}\ln(3x + 2)$

$y' = \frac{1}{2} \cdot \frac{3}{3x+2} = \frac{3}{2(3x+2)}$

5) $y = \ln(\frac{3x-5}{2x-1})$ $\quad y = \ln(3x - 5) - \ln(2x - 1)$

$y' = \frac{3}{3x-5} - \frac{2}{2x-1}$

6) $y = \ln(4x^3)$ $\quad y' = \frac{12x^2}{4x^3} = \frac{3}{x}$

7) $y = \ln(3x^2\sqrt{5x + 2})$ $\quad y = \ln(3x^2) + \ln(5x + 2)^{1/2} = \ln(3x^2) + \frac{1}{2}\ln(5x + 2)$

$y' = \frac{6x}{3x^2} + \frac{1}{2} \cdot \frac{5}{5x+2} = \frac{2}{x} + \frac{5}{2(5x+2)}$

Delta Ex 7.2, Ex 7.3 pg 88
Extension Ex 7 pg 90